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Detecting matter effects in long baseline experiments

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Abstract: Experiments strongly suggest that the flavour mixing responsible for the atmospheric neutrino anomaly is very close to being maximal. Thus, it is of great theoretical as well as experimental importance to measure any possible deviation from maximality. In this context, we reexamine the effects of matter interactions in long baseline neutrino oscillation experiments. Contrary to popular belief, the muon neutrino survival probability is shown to be quite sensitive to matter effects. Moreover, for moderately long baselines, the difference between the survival probilities for ν_μ and $\bar{\nu}_\mu$ is shown to be large and sensitive to the deviation of $|U_{\mu 3}|$ from maximality. Performing a realistic analysis, we demonstrate that a muon-storage ring ν-source alongwith an iron calorimeter detector can measure such deviations. (Contrary to recent claims, this is not so for the NuMI-minos experiment.) We also discuss the possible correlation in measuring $U_{\mu 3}$ and U_{e3} in such experiment.

Keywords: Neutrino Physics, Standard Model.

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Contents

1. Introduction

Neutrino masses and mixings continue to intrigue us, despite the continued efforts of many recent experiments [1 − 6]. We do know that the muon-neutrino (ν_μ) mixes almost maximally ($\sin^2 2\theta_{\mu 3} > 0.92$ at 90% C.L) with another species ν_3 (which could be identified with the tau-neutrino, ν_τ) and that the mass splitting $|\delta m^2_{32}| \sim 2 \times 10^{-3}\,\mathrm{eV}^2$. Furthermore, the electron-neutrino (ν_e) mixes with a combination of ν_μ and ν_τ with a similarly large angle $\theta_{e2} \simeq 30^\circ$ but a far smaller mass splitting ($\delta m^2_{21} \sim 7 \times 10^{-5}\,\mathrm{eV}^2$). And finally, the third mixing angle θ_{13} is constrained, by the CHOOZ experiment [3], to be small. What we do not know though are the sign of δm^2_{32}, the precise value of θ_{13} and the extent to which $\theta_{\mu 3}$ may, if at all, differ from maximality. Since such knowledge is obviously essential to the formulation of a theory of neutrino flavours, the importance of precise determinations cannot but be underemphasised. Neutrino oscillations provide just the proper platform for such an exercise.

As is well known, during passage through dense matter, various amplitudes for neutrino oscillations may be magnified to a great degree [7] thereby rendering them measurable with relative ease. While propagating in a medium, all the three neutrino flavours interact with matter via neutral current (NC) interactions with equal strength. Those of the electron flavour, $\overset{(-)}{\nu}_e$, have, in addition, a extra charged current interaction as well. Since it is this extra interaction that provides *flavour-dependence*, all matter-induced effects in neutrino oscillations (including those in the ν_μ-ν_τ sector) are proportional to the size of the relevant mixing with ν_e.

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Recent theoretical advances in the quest of producing very high intensity muon sources engender optimism regarding a future neutrino factory wherein an intense beam of muons is to be accelerated to a not too high energy and stored in a storage ring with a straight section directed towards a neutrino detector. Muons would decay in this straight section thereby producing high intensity neutrino beams (both of electron- and muon-types) that are highly collimated in the direction of the decaying muons [8]. The efficacy of muon storage ring neutrino sources in performing precision measurements of neutrino parameters has already been discussed in the literature [9, 10]. The advantages over neutrino experiments with conventional neutrino beams (arising from $\pi^{\pm}$ decay) are manifold: (*i*) a precise knowledge of the ν_μ and $\bar{\nu}_e$ fluxes helps reduce the systematic errors; (*ii*) assuming a μ-beam of, say, 20 GeV energy, such neutrinos have, on the average, energies higher than those of conventional neutrino beams thereby increasing the ν cross-section at the detector; and (*iii*) the aforementioned collimation.

Previous studies of the matter effect at a long base line experiment [11, 12] were concentrated mainly on the measurement of the rate of wrong sign muon events, which, in turn, is proportional to the transition probability $P_{\nu_e \to \nu_\mu}$. Such a measurement would enable a determination of (or constraining of) $\sin\theta_{13}$ *provided* independent and precise measurements of the mixing angle θ_{23} and mass-square difference δm^2_{31} exist. Possible measurements of neutrino mixing parameters from long baseline experiments using conventional neutrino beams as well as from a neutrino factory, and a water Čerenkov detector have also been discussed in ref. [13].

In the present article, we demonstrate that a sizable matter effect is also evinced by a measurement of the ν_μ survival probability. While this had been remarked on in ref. [10], we go significantly beyond in our analysis. In addition, such a measurement has the potential for detecting possible deviations from maximality of the mixing angle responsible for the atmospheric neutrino anomaly. Correlation of matter effects and the aforementioned maximality has been investigated in the context of long-baseline experiments [14], and more recently, also in the context of future atmospheric neutrino experiments [15]. In [16], the presence of large matter effects in $\nu_\mu - \nu_\tau$ oscillation as well as in ν_μ survival probability has been stressed. However, even though a large matter effect is present in the $\nu_\mu - \nu_\tau$ sector, measurement of the same is difficult due to both the lower detection efficiency for the tau-events as well as the need for a more sophisticated detector.

Whereas much of our analysis would be independent of the exact nature of the source as well as the detector, we also choose to examine the feasibility of making such measurements in the context of a realistic experimental setup. To be concrete, we choose a 50 kT Iron detector, with detection and charge discrimination capability for muons provided by a magnetic field. Such a detector was proposed for Gran Sasso (MONOLITH) [17] and, more recently, for a location in India (INO) [18]. The latter is contemplated primarily as a detector for atmospheric neutrinos and also as an end detector for a future neutrino factory beam. A muon detection threshold of 2 GeV [18] has been used in our calculation. Such a treatment implicitly assumes an ideal situation wherein the detector response is just a step function of the incident muon energy. In real life, of course, such would not be the case. It must be borne in mind, though, that a very realistic estimation of such effects is only possi-

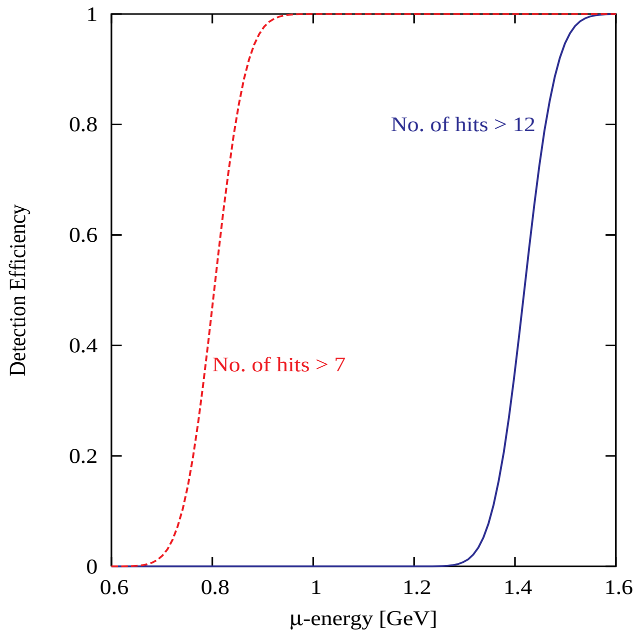

Figure 1: Muon detection efficiency as a function of muon energy. The solid line corresponds to muon identification with 7 hits in the detector according to INO/ICAL simulation. We also present the detection efficiency if 12 hits are necessary to detect a muon.

ble in the context of a particular detector. Since experiments such as the MONOLITH [17] or the ICAL/INO [18] are still very much in the future, our comments can be based only on the results of the detector simulations and prototype studies effected by these collaborations.

The threshold for the charged-current interaction is determined by the number of hits the emergent muon makes in the detector. The latter is dependent not only upon the energy and angle of the muon, but also on the geometry of the detector (alongwith the position of the initial interaction point within the detector) as well as the number density of the active detector components. The ICAL/INO collaboration has effected detailed detector simulations [19] and find that the number of hits that the emergent muon results in is well approximated by a gaussian distribution with the mean being proportional to the muon energy and the width virtually independent of it. Accepting the collaboration's benchmark of *seven* hits in the detector to constitute an observation, we present, in figure 1, the resulting detector response as a function of the incident muon energy. From the figure it is evident that for an incident muon energy of larger than approximately 1 GeV, the probability of muon detection is practically unity. On the other hand, if we make a more stringent demand of a minimum of 12 hits in the detector to constitute an observation, the threshold is shifted to approximately 1.5 GeV. This justifies our assumption about the energy threshold (2 GeV) we have used in our analysis.

As for the baseline, we illustrate our results for two particular choices. The first one corresponds to a distance from JHF to INO ($\sim$ 5000 Km) and the other one corresponds to an even longer baseline, namely from FERMILAB to INO ($\sim$ 10000 Km). We will also compare results with a baseline of 732 Km, corresponding to the Fermilab-MINOS distance.

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The plan for the rest of the article is as follows. In section 2, we discuss neutrino oscillation in the presence of matter. We also define the asymmetry between μ^+ and μ^- events and explain its correlation to U_{e3} and $U_{\mu 3}$. We present the expressions for the asymmetry for two different approximations: when matter effect is small compared to $\delta m^2_{32}/2E_\nu$ i.e for short base line experiments like MINOS [20] and also when the matter term is comparable to the $\delta m^2_{32}/2E_\nu$ i.e., for long base line experiments. In section 3, we briefly review and reexamine the proposal, in a recent article [20], to measure matter effect and the above correlations in the context of the MINOS experiment. Section 4 deals with the expectations at long baseline experiments, both for the ν_μ survival probability as well as for the ν_e to ν_μ transition probability. An analysis of the findings in the context of a realistic experimental situation is effected in section 5. Finally, we conclude in section 6.

2. Neutrino oscillation in presence of matter

We begin by reviewing the passage of neutrino through matter. While analytical results are possible, the exact answers are cumbersome and not very useful. However, it is useful to consider approximations such as a constant density profile for the earth. Several approximate expressions for oscillation probability in the presence of matter are available in the literature [21, 12]. While such an assumption is obviously not a very accurate one, especially when neutrinos traverse a long distance through the earth, it, nevertheless serves to provide us some physical insights into the problem we are dealing with. Furthermore, results such as those we shall obtain are still applicable if the matter density within the earth may be approximated to be piecewise constant. It must be kept in mind though that the quantitative results presented in this article are obtained not by means of such an approximation, but by explicitly accounting for the varying density profile of the earth using the Preliminary Reference Earth Model (PREM) [22].

As with the quark sector, neutrino flavour states can be expressed in terms of mass eigenstates through a relation of the form

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}\,|\nu_i\rangle\,, \tag{2.1}$$

where U is a 3×3 unitary matrix known as the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix. In the absence of Majorana phases (assumed to be so henceforth),[1] U can be parametrized in terms of three mixing angles and a CP-violating phase $\delta_{\rm CP}$, *viz.*

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta_{\rm CP}} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta_{\rm CP}} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix}, \tag{2.2}$$

with $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. In vacuum, neutrino oscillations are then governed by U and the two independent differences in the squares of the neutrino masses, $\delta m^2_{ij} \equiv m^2_i - m^2_j$. In the rest of our analysis, we will make a further simplifying assumption of $\delta_{\rm CP} = 0$. With the above parametrization of U, experimental results can be used to constrain the mixing

[1] Any Majorana phase in the PMNS matrix cannot be probed by an oscillation experiment.

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angles θ_{ij} and the mass-square differences. Measurements of the solar neutrino fluxes, augmented by the KamLAND reactor data give

$$0.30 < \tan^2\theta_{12} < 0.64\,, \qquad 5.4\times 10^{-5}\,\mathrm{eV}^2 < \delta_{21} < 9.4\times 10^{-5}\,\mathrm{eV}^2 \tag{2.3}$$

with the best-fit values given by $\tan^2\theta_{12} = 0.40$ and $\delta m^2_{21} = 6.9\times 10^{-5}\,\mathrm{eV}^2$. Experiments with atmospheric neutrinos (as well as the K2K experiment) suggest, on the other hand, that

$$\sin^2 2\theta_{23} > 0.86\,, \qquad 1.4\times 10^{-3}\,\mathrm{eV}^2 < |\delta m^2_{32}| < 5.1\times 10^{-3}\,\mathrm{eV}^2\,, \tag{2.4}$$

with the best-fit value being $|\delta m^2_{32}| = 2.0\times 10^{-3}$ eV^2. And, finally, the negative result of the CHOOZ experiment implies that

$$\sin^2\theta_{13} < 5\times 10^{-2} \qquad \text{at } 99.73\%\ \text{C.L.} \tag{2.5}$$

Note that,

- unlike in the case for δm^2_{21}, the sign of δm^2_{32} is undetermined;
- since $|\delta m^2_{32}| \approx |\delta m^2_{31}| \gg |\delta m^2_{21}|$, the last-mentioned difference (as well as the solar mixing angle θ_{12}) plays only a subservient role in ν_μ oscillations;
- while $|U_{\mu 3}|$ is constrained to be close to $1/\sqrt{2}$, it is still allowed to be non-maximal.

Working in the limit of a vanishingly small δm^2_{21}, all the expressions for the oscillation/survival probabilities can be expressed in terms of the dominant mass-square difference by (which we henceforth denote by δm^2_{31}) and just two of the mixing angles,[2] *viz.* θ_{23} and θ_{13}. For a ν_μ ($\bar\nu_\mu$) of energy E_ν traveling a distance L in vacuum, the survival probability is then given by

$$P^{\rm vac}_{\mu\mu} = P^{\rm vac}_{\bar\mu\bar\mu} = 1 - 4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2)\sin^2\left(\frac{\Delta_{31}L}{2}\right), \tag{2.6}$$

where,

$$U_{e3} = s_{13}\,, \qquad U_{\mu 3} = c_{13}\,s_{23}\,, \qquad \Delta_{31} \equiv \frac{\delta m^2_{31}}{2E_\nu}\,. \tag{2.7}$$

In the presence of matter, the electron (anti-)neutrino sees an additional potential corresponding to charged current interactions, and the effective hamiltonian is given by

$$H = U\begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 \\ 0 & 0 & \Delta_{31}\end{pmatrix}U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{pmatrix}, \tag{2.8}$$

where $A \equiv \pm\sqrt{2}G_F N_e$ for $\overset{(-)}{\nu}_e$, with N_e being the instantaneous electron density of the matter.

[2]While the analytical expressions below have been derived under this approximation, all of the numerical analysis has been performed using the full expression for U and a non-zero value for δ_{12}.

The survival probabilities suffer a consequent modification. While the full expressions are cumbersome, it is instructive to look at a simplified form obtainable for a matter density small enough that A can be treated as a perturbation in the effective hamiltonian. In this limit, following ref. [20],

$$\begin{aligned} P_{\mu\mu\,(\bar\mu\bar\mu)} &\simeq P^{\rm vac} \pm \left(\frac{\Delta P_{\mu\mu}}{2}\right) + \mathcal{O}(\widetilde{A}^2) \\ \Delta P_{\mu\mu} &\simeq 4\,\widetilde{A}\,|U_{e3}|^2|U_{\mu 3}|^2(1-2|U_{\mu 3}|^2)\left[4\sin^2\Psi - \Psi\,\sin(2\,\Psi)\right] + \mathcal{O}(\widetilde{A}^2) \end{aligned} \tag{2.9}$$

where

$$\widetilde{A} \equiv \frac{A}{\Delta_{31}} \qquad \text{and} \qquad \Psi \equiv \frac{\Delta_{31}\,L}{2}\,. \tag{2.10}$$

The probability difference[3] $\Delta P_{\mu\mu}$ is a measure of the matter effect and that it should be proportional to both $\widetilde{A}$ and $|U_{\mu 3}|^2$ is obvious. Note also that the ν_μ has the same interaction with matter as the ν_τ with which it primarily mixes and any matter effect can only seep in through a mixing with the ν_e. This, then, accounts for the overall factor of $|U_{e3}|^2$. All of these three proportionalities quite independent of the approximation, and, in fact, are exact results. The factor $(1-2|U_{\mu 3}|^2)$, however, is but a consequence of the approximation of a small matter term $\widetilde{A}$ and is applicable only for neutrinos traversing small base lines. Since this factor vanishes identically for a maximal mixing in the $\nu_\mu - \nu_\tau$ sector, this matter-induced asymmetry is potentially a sensitive probe of the maximality of $U_{\mu 3}$. This has already been pointed out [20] and examined in the context of the MINOS experiment. In the next section we will briefly reexamine their claims.

Convoluting eq. (2.9) with the E_ν-dependent flux and the detection efficiency of a given detector, one would obtain the number of events and thereby the event asymmetry

$$\Delta N = N_{\nu_\mu} - N_{\bar\nu_\mu} \equiv N_- - N_+\,. \tag{2.11}$$

The latter could then be used to place constraints in the $|U_{\mu 3}|$-$|U_{e3}|$ plane. Of course, in a realistic case, the effects of a varying matter density has to be taken into account as well and this we do include in our analysis. With the effect being proportional to $\widetilde{A}$, it will be magnified when the (anti-)neutrino beam traverses regions of high density, or in other words, the core of the earth. Note, however, that a large $\widetilde{A}$ results in a breakdown of the simplified form given in eq. (2.9) and that the full expression (or, at least, a different approximation) needs to be used.

The exact expression (for an arbitrarily large but constant density) for the survival probability is given by

$$\begin{aligned} P_{\mu\mu} = 1 - 4\,s_{23}^2\Bigg(& c_{23}^2 s_{\theta_m}^2 \sin^2\left[\frac{\Delta_{31}L}{4}\,(1+\widetilde{A}-\mathcal{D})\right] + \\ & + s_{23}^2 s_{\theta_m}^2 c_{\theta_m}^2 \sin^2\frac{\Delta_{31}L\mathcal{D}}{2} + c_{23}^2 c_{\theta_m}^2 \sin^2\left[\frac{\Delta_{31}L}{4}(1+\widetilde{A}+\mathcal{D})\right]\Bigg)\,, \end{aligned} \tag{2.12}$$

[3] In a sense, $\Delta P_{\mu\mu}$ is the **asymmetry** between the effect that the “matter” medium has on the particle (ν) and antiparticle ($\bar\nu$) propagation.

where

$$\mathcal{D} \equiv \sqrt{1+\widetilde{A}^2 - 2\,\widetilde{A}\cos 2\theta_{13}}\,, \qquad \text{and} \qquad \theta_m \equiv \frac{1}{2}\tan^{-1}\frac{\sin 2\theta_{13}}{\cos 2\theta_{13} - \widetilde{A}}\,. \tag{2.13}$$

We now perform a double expansion[4] in $|U_{e3}|$ as well as $\beta \equiv \frac{1}{2} - |U_{\mu 3}|^2$ while allowing for any value for $\widetilde{A}$. Since both the expansion parameters are small, we may retain terms only upto, say, the fourth order without any loss of accuracy. Then, for the survival probability, we have

$$\begin{aligned}
P_{\mu\mu} = {} & (1-4\,\beta^2)c^2_\Psi + \\
& + \frac{|U_{e3}|^2}{(1-\widetilde{A})^2}\Big[\Big\{s^2_\Psi - s^2_\Omega - s^2_\Theta - \widetilde{A}(1-\widetilde{A})\Psi\, s_{2\Psi}\Big\} - 4\beta\Big\{(1-\widetilde{A})^2\, s^2_\Psi - s^2_\Omega\Big\} + \\
& \qquad + 4\,\beta^2\Big\{(1-4\widetilde{A}+2\widetilde{A}^2)\, s^2_\Psi + s^2_\Omega - s^2_\Theta + \frac{\widetilde{A}}{2}(1-\widetilde{A})\Psi\, s_{2\Psi}\Big\}\Big] + \\
& + \frac{|U_{e3}|^4}{(1-\widetilde{A})^4}\Big[\Big(1-6\,\widetilde{A}+\widetilde{A}^4-4\,\widetilde{A}^3+5\widetilde{A}^2\Big)\, s^2_\Psi + (2\,\widetilde{A}+\widetilde{A}^2)\, s^2_\Omega - \\
& \qquad - \Omega^2\,(1-\widetilde{A})^2\, c_{2\Psi} + (-1+6\,\widetilde{A}-\widetilde{A}^2)\, s^2_\Theta + \\
& \qquad + (1-\widetilde{A}^2)\Omega\, s_{2\Psi} + (1-\widetilde{A})\Omega(s_{2\Omega} + 2\, s_{2\Theta})\Big] + \\
& + \mathcal{O}(|U_{e3}|^4\beta, |U_{e3}|^3\beta^2, |U_{e3}|^2\beta^3, |U_{e3}|\beta^4)\,,
\end{aligned} \tag{2.14}$$

where $s_\alpha \equiv \sin(\alpha)$, $c_\alpha \equiv \cos(\alpha)$ with

$$\Omega \equiv \frac{\Delta_{31}\, L\,\widetilde{A}}{2} = \frac{A\,L}{2} = \widetilde{A}\,\Psi \qquad \text{and} \qquad \Theta \equiv \Omega - \Psi\,. \tag{2.15}$$

With the survival probability for the antineutrinos being obtained by the replacement $\widetilde{A} \to -\widetilde{A}$ in eq. (2.14), we have, for the difference in the survival probabilities (for ν_μ and $\bar{\nu}_\mu$):

$$\begin{aligned}
\frac{-(1-\widetilde{A}^2)^3\,\Delta P}{4\,\widetilde{A}\,|U_{e3}|^2\,|U_{\mu 3}|^2} = {} & (1-\widetilde{A}^2)\Big[4\,c^2_\Psi\, s^2_\Omega + (1-\widetilde{A}^2)\,\Psi\, s_{2\Psi} - \frac{1+\widetilde{A}^2}{2\,\widetilde{A}}\, s_{2\Psi}\, s_{2\Omega}\Big] + \\
& + 2\,\beta\,(1-\widetilde{A}^2)\,\Big[-4\, s^2_\Psi\, s^2_\Omega + (1-\widetilde{A}^2)\,\Psi\, s_{2\Psi} + \frac{1+\widetilde{A}^2}{2\,\widetilde{A}}\, s_{2\Psi}\, s_{2\Omega}\Big] + \\
& + |U_{e3}|^2\Big[\frac{4}{1-\widetilde{A}^2}\,\Big\{(1+11\,\widetilde{A}^2+2\,\widetilde{A}^4)\, s^2_\Psi\, c^2_\Omega + \widetilde{A}^2\,(3-\widetilde{A}^2)\, c^2_\Psi\, s^2_\Omega\Big\} + \\
& \qquad + \Psi\Big\{2\,\widetilde{A}^2\,(1-\widetilde{A}^2)^2\,\Psi\, c_{2\Psi} - (1+6\,\widetilde{A}^2+\widetilde{A}^4)\, s_\Psi\, c_\Psi\Big\} + \\
& \qquad + s_{2\Psi}\Big\{2\,(1+3\,\widetilde{A}^2)\Psi\, c_{2\Omega} - \frac{1-16\,\widetilde{A}^2-\widetilde{A}^4}{4\,\widetilde{A}}\, s_{2\Omega}\Big\} - \\
& \qquad - (3+\widetilde{A}^2)\,\Omega\, s_{2\Omega}(3-4\, s^2_\Psi)\Big] + \\
& + \mathcal{O}(|U_{e3}|^2\beta, |U_{e3}|\beta^2, \beta^3)\,.
\end{aligned} \tag{2.16}$$

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[4]This is similar in spirit to, though not the same as, the expansion performed in ref. [23].

Note that

- The probability asymmetry ΔP continues to be proportional to each of $\widetilde{A}$, $|U_{e3}|^2$ and $|U_{\mu 3}|^2$, but not to $(1 - 2|U_{\mu 3}|^2)$.
- $\lim_{\widetilde{A}\to 0} s_\Omega/\widetilde{A} = \Psi$ and hence the right hand side of eq. (2.16) is finite in this limit.
- To $\mathcal{O}(\widetilde{A})$, the expressions for both $P_{\mu\mu}$ and ΔP coincide with those of eq. (2.9).
- The strength of the $\mathcal{O}(1)$ term on the r.h.s. of eq. (2.16) — which vanishes as $\widetilde{A}^2$ for small $\widetilde{A}$ — gives a measure of the violation of the $(1 - 2|U_{\mu 3}|^2)$ proportionality of ΔP.
- While we have performed the expansion upto $\mathcal{O}(|U_{e3}|^4)$, for all practical purposes it suffices to consider only upto $\mathcal{O}(|U_{e3}|^2)$.
- The apparent singularities in eqs. (2.14 & 2.16) are not real ones, for

$$\begin{aligned}\lim_{\widetilde{A}\to 1} P_{\mu\mu} = c_\Psi^2 + 4\,\beta^2\, s_\Psi^2 + |U_{e3}|^2 \Big[\Psi\,(s_{2\Psi} - 2\,\Psi\, c_\Psi^2) + 4\,\beta(\Psi^2 - s_\Psi^2) + \\ + 4\,\beta^2\{2(1-\Psi^2)s_\Psi^2 - \Psi\, s_{2\Psi}\}\Big] + \\ + |U_{e3}|^4\,\Psi^2 \left[s_\Psi^2 + \frac{\Psi^2}{3}\,(3 + 2\,s_\Psi^2) - \frac{4}{3}\,\Psi\, s_{2\Psi}\right],\end{aligned} \tag{2.17}$$

and

$$\begin{aligned}\lim_{\widetilde{A}\to 1} \frac{-\Delta P}{|U_{e3}|^2\,|U_{\mu 3}|^2} = 2\left[2\,\Psi^2\, c_\Psi^2 - \Psi\, s_\Psi\, c_\Psi - \frac{s_{2\Psi}^2}{4}\right] + \\ + \beta\Big[-\Psi\, s_{2\Psi} + (5 - 8\,\Psi^2)\, s_\Psi^2 + s_{2\Psi}^2\Big] + \\ + |U_{e3}|^2\left[-s_{2\Psi}^2 - \Psi^2\left(\frac{3}{2} + s_\Psi^2\right) - \frac{2\,\Psi^4}{3}(3 + 2\,c_\Omega^2) + \right. \\ \left. + \frac{4\,\Psi^3}{3}\, s_{2\Psi} + \Psi\, s_{2\Psi}\left(\frac{5}{8} - s_\Psi^2\right)\right].\end{aligned} \tag{2.18}$$

3. Can the NuMI-MINOS combine see this?

With the Fermilab Main Injector serving as a neutrino source, the MINOS detector [24] at the Soudan Mine offers a baseline of 732 km and hence a possible setup for making such a measurement. This was investigated by Choubey and Roy in ref. [20]. Working with a ν_μ exposure of 16×10^{20} primary protons on target, they conclude that a significant rate asymmetry (ΔN) can be observed for moderate values of $(1 - 2|U_{\mu 3}|^2)$ and $|U_{e3}|^2$.

However, while their idea was an interesting one and the formalism essentially correct, a re-estimation of the rate asymmetry using the same set of inputs reveals some discrepancies. While we agree with ref. [20] on the magnitude of ΔN for all the different cases of inputs parameters, we completely disagree on the issue of the error bars. Even on using an optimistic indicator of the background fluctuation, namely $\sqrt{N_\mu + N_{\bar{\mu}}}$, we find that, for

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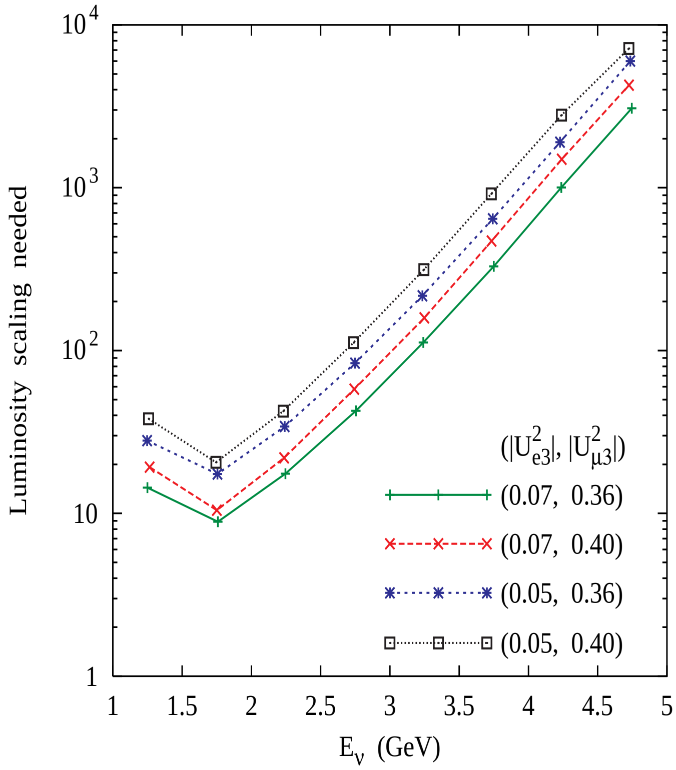

Figure 2: The minimal ratio by which the MINOS ν_μ exposure (16×10^{20} primary protons on target) needs to be scaled up to obtain 1σ effects for each individual energy bin. The curves correspond to the four parameter points of ref. [20]. Note that only the statistical fluctuations have been included in this analysis.

the above mentioned luminosity, the statistical error bars are large enough for each of the rate asymmetries in ref. [20] to be consistent with zero.

Thus, to obtain a result of any statistical significance would need the flux (or, equivalently, the detector size) to be increased manifold (see figure 2). Such a large scaling, unfortunately, is not feasible with the present experimental facility. Note, in addition, that this projection altogether neglects the systematic uncertainties, and consequently is already too optimistic. In reality, several uncertainties are involved in the translation between the visible energy observed at a detector and the energy of the neutrino to which the event is ascribed. These include the uncertainties in the assumed final state multiplicities, the scattering or absorption of the secondary particles etc. In a recent paper [25], the MINERνA collaboration, estimates that the size of the corresponding systematic effects at MINOS-like experiments could be comparable to, or even dominate, the statistical errors.

4. The case for a very long baseline

Having established that, contrary to the claims of ref. [20], MINOS, in its current incarnation, is not sensitive to possible deviations of $|U_{\mu3}|$ from maximality even for optimistic choices of parameters, it becomes interesting to investigate if any of the other proposed long-baseline

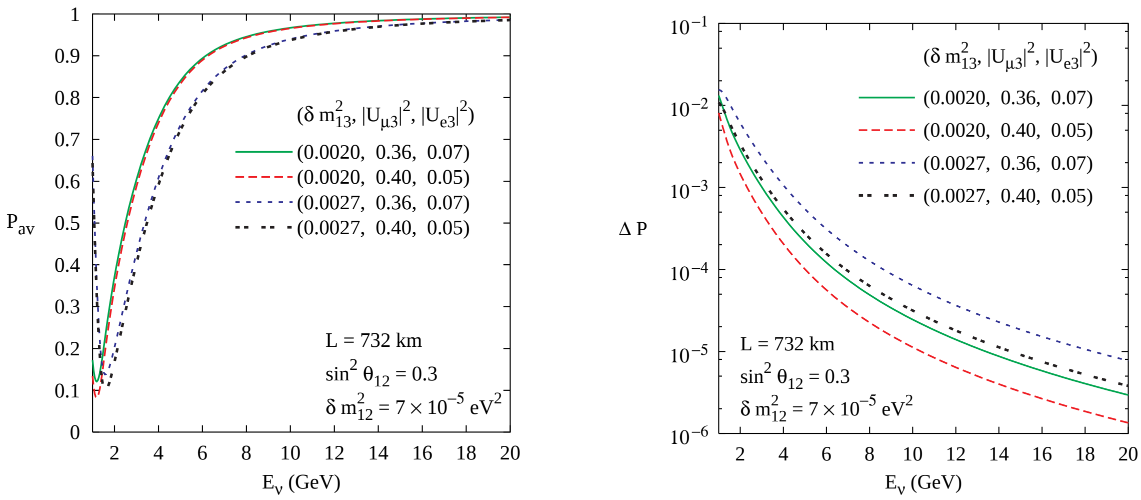

Figure 3: *(a)* The neutrino survival probability as a function of neutrino energy for a baseline $L = 732$ km. The curves correspond to different combinations of δm^2_{13} (in eV^2), $|U_{\mu 3}|^2$ and $|U_{e3}|^2$. *(b)* The corresponding difference in the ν_μ and $\bar{\nu}_\mu$ survival probabilities.

experiments would do the job. And, in a similar vein, if even the luminosity scaling as described in the previous section would prove adequate for the NuMI-MINOS combine. We start by investigating the second problem.

As has already been emphasised above, it is not only ΔP that is important, but also the average survival probability $P_{\rm av} \equiv (P_{\mu\mu} + P_{\bar{\mu}\bar{\mu}})/2$ for the latter controls the total number of events and hence the size of the statistical error (significance $\propto \Delta P_{\mu\mu}/\sqrt{2P_{\rm av}}$). In figure 3, we present both these quantities as function of neutrino energy for a baseline of 732 km. The two choices of $(U_{\mu 3}|^2, |U_{e3}|^2)$ correspond to the most (and least) optimistic sets of parameters used in ref. [20].

It is abundantly clear that, for such a baseline, and for realistic choice of parameters, ΔP is very small, especially at larger neutrino energies. This was to be expected as a baseline of 732 km implies that the neutrino beam neither travels through a very dense matter core nor does it travel through a "dilute" segment long enough for the matter effect to build up sufficiently. Clearly, it is dangerous to claim a signal based on $\Delta P \lesssim 10^{-4}$, for such an act presupposes a very accurate knowledge of both the (anti-)neutrino cross sections as well as the density profile within the earth. Thus, one would have to concentrate only on neutrinos with relatively small energies ($E_\nu \lesssim 2\,$GeV). Apart from the fact that the asymmetry is not too large even in this range,[5] such a measurement also necessitates both a very good energy resolution as well as a low energy threshold. Furthermore, for such small energies, neutrino cross-section scales as E_ν^2 and thus restricting ourselves to a small window is tantamount to rejecting a very large fraction of the events.

The situation, vis. a vis. the size of ΔP, seems to improve dramatically with an increase in the baseline. For definiteness, we consider two particular values, namely $L = 4865$ km (the distance between the Japan Hadron Facility and the proposed Indian Neutrino

[5]In fact, it stands to be swamped by the systematic errors unless further experiments are performed [25].

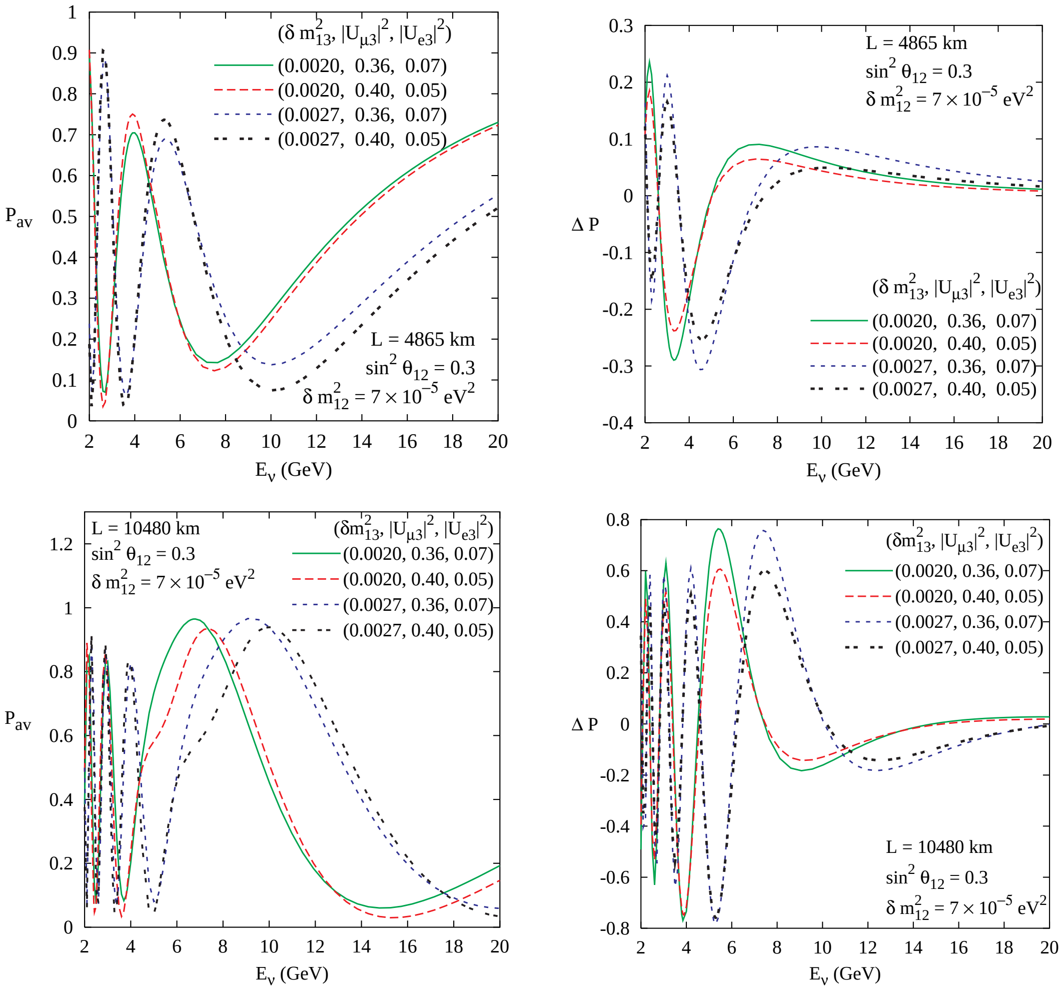

Figure 4: As in figure 3, but for baselines $L = 4865$ km (upper panels) and $L = 10480$ km (lower panels) instead.

Observatory site at Rammam [18]) and $L = 10480$ km (the distance between Fermilab and INO). Since the matter term can no longer be treated as a perturbation, we desist from using the simplified form of eq.(2.9) and use the full expression for neutrino propagation in matter with the density profile being given by the Preliminary Reference Earth Model [22]. As figure 4 amply demonstrates, for each of these values, ΔP is large over a wide range of E_ν. In fact, for the larger of the two baselines $|\Delta P|$ can be as large as 0.8 for certain E_ν bins. Furthermore, unlike in the case of the 732 km baseline where ΔP was nearly monotonic in E_ν, significant structure, including sign reversal, is shown. As can be easily appreciated, the latter feature has the potential of serving as a key experimental signature.

A further point of interest relates to the position of the maxima of the survival probabilities as well as the difference ΔP. Clearly, a high resolution measurement of the same could, in principle, be used to determine δm^2_{13}.

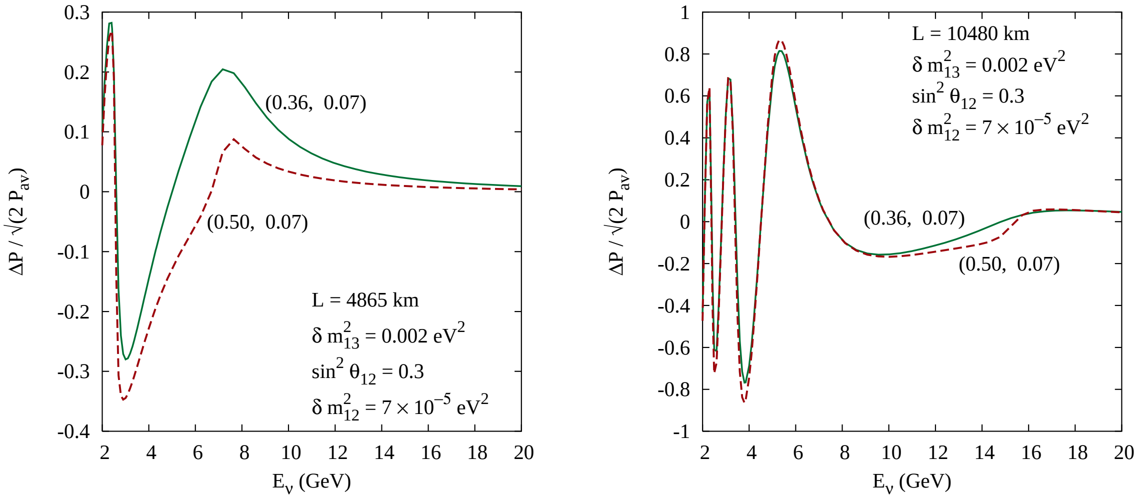

Figure 5: The normalized probability difference for a baseline of *(a)* 4865 km and *(b)* 10480 km. In each case, the solid (dashed) line refers to $|U_{\mu 3}|^2 = 0.36$ (0.50). Note that, according to the leading expression of eq.(2.9), the dashed curves should have coincided with the abscissa.

Before we interpret figure 4 to imply that the asymmetry measurement in very long baseline experiments would be a sensitive probe of the deviation of $|U_{\mu 3}|$ from maximality, it should be realised that the latter is not the only source for a non-zero asymmetry. In fact, as has already been hinted at above, the higher order terms in $\widetilde{A}$ that were dropped while deriving eq. (2.9) are *not* proportional to $(1 - 2|U_{\mu 3}|^2)$. This can also be seen explicitly from eqs. (2.14 & 2.16) obtained assuming a constant density profile for earth. The numerical importance of such contributions naturally increases with the baseline, and has been explicitly displayed in figure 5. In fact, for the larger baseline, it is quite apparent that the bulk of the effect is due to such "higher-order" terms. Thus, it may be concluded that while a baseline of ≈ 5000 km may allow for a determination of $(1 - 2|U_{\mu 3}|^2)$ through measurements of the rate asymmetry, for baselines much longer than this the sensitivity reduces quite sharply, and a measurement of the maximality of $|U_{\mu 3}|$ is not very straightforward for ultra-long baselines.

On the other hand, the asymmetry must remain proportional to $|U_{e3}|^2$ even on the inclusion of $\widetilde{A}^n$ terms, for it is only through the interaction with the ν_e that matter effect truly enters ν_μ-ν_τ oscillations. Thus, a baseline of 10480 km may still serve the purpose of affording a good measurement of $|U_{e3}|$. In fact, since $U_{\mu 3}$ and U_{e3} are inextricably linked even for the case of the 4865 km baseline, such an independent measurement of the latter element has its own importance.

4.1 $\nu_e \to \nu_\mu$ oscillations

With the matter effect playing such an important role in the survival probability for the muon (anti-)neutrino, it becomes interesting to consider its effect on $P(\nu_e \to \nu_\mu)$. For, a source that produces ν_μ $(\bar{\nu}_\mu)$ copiously, whether it be a beam dump or a muon storage ring, would also produce electron (anti-)neutrinos.

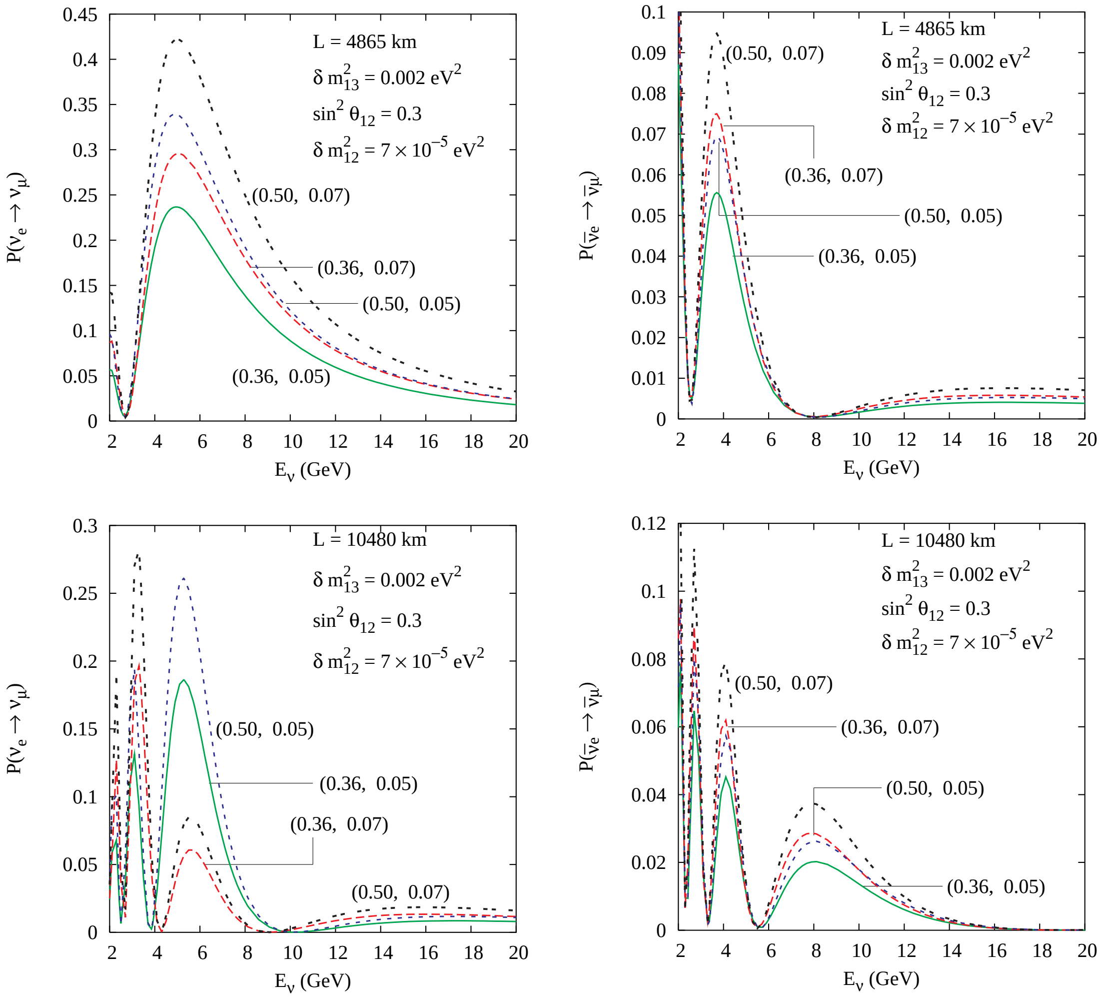

Figure 6: The energy dependence of the transition probability for $\nu_e \to \nu_\mu$ (left) and $\bar{\nu}_e \to \bar{\nu}_\mu$ (right) on passage through matter. The upper and lower panels correspond to baselines of 4865 and 10480 km respectively.

In figure 6, we present the probabilities for such transitions as a function of (anti-)neutrino energy. Several observations are in order:

- The transition probability $P(\nu_e \to \nu_\mu)$ can be quite sizable for parameter values that are still allowed by experimental data.
- For $E \gtrsim 3\,\text{GeV}$, we see that $P(\bar{\nu}_e \to \bar{\nu}_\mu) \lesssim P(\nu_e \to \nu_\mu)$ (except, of course, near the node at $\sim 8\,\text{GeV}$ for the 10480 km case). Coupled with the smaller detection efficiency for anti-neutrinos, this indicates that it would be more profitable to work with ν_e than $\bar{\nu}_e$.
- The transition probabilities have non-trivial dependence on both U_{e3} and $U_{\mu 3}$. Unlike in the case of the aforementioned asymmetry, these, in general, are *not* proportional to $|U_{e3}|^2$.

- The measurement of this effect would thus lead to a constraint in the U_{e3}-$U_{\mu 3}$ plane independent of that drawn from the asymmetry measurement.

5. Event rate calculation from a storage ring neutrino source

As we have already mentioned in the introductory section, the neutrino flux from a muon storage ring can be calculated very precisely. Starting with a μ^- beam, the number of μ^- events in a far detector can be obtained by folding this flux with the survival probability $P_{\mu\mu}(L, E)$ and the charged current cross-section:

$$N_\mu = N_n \int \sigma(\nu_\mu + N \to \mu^- + X) \, \frac{dN_\nu}{dE_{\nu_\mu}} \, P_{\mu\mu}(L, E) \, dE_{\nu_\mu} \tag{5.1}$$

where N_n is the total number of nucleons present in the fixed target. An analogous expression obtains for μ^+ events as well. Note though that the $\bar{\nu}_e$ from muon decay could also oscillate into $\bar{\nu}_\mu$ while traversing through the earth and result in muonic events. However, they result in *wrong sign* muons and thus can be easily distinguished in a magnetized detector. We shall revert back to them at a later stage.

To be specific, we shall consider a storage ring with 20 GeV muons and a 50 kT Iron calorimeter detector such as the one proposed for MONOLITH [17] or the ICAL/INO [18] experiments. The projected energy threshold for muon detection is about 2 GeV and the resolution is expected to be better than 0.5 GeV over the entire range.

Before delving into the actual event profile, several points are in order.

- As has already been demonstrated in the previous section, the difference in the survival probabilities ($\Delta P_{\mu\mu}$) typically grows with the baseline. Thus the experimental sensitivity could be expected to increase with the baseline. (This is modulo the fact that the sensitivity of the asymmetry to $(1-2\,|U_{\mu 3}|^2)$ reduces for very large baselines, while the sensitivity to $|U_{e3}|^2$ is retained.)

- On the other hand, for a fixed detector size, the effective solid angle subtended by the detector to the source decreases with increasing baseline. Consequently the neutrino flux goes down quadratically with the baseline, thereby resulting in a smaller number of events and hence larger statistical errors. This, for example, can be read off from figure 7, wherein we have displayed the number of events expected if the survival probability were to be unity.

- One obvious way to circumvent the latter problem is to start with higher luminosity. For our numerical results we then adopt the 'higher luminosity' option for the muon beam, namely 10^{20} muon decays per year.

- An alternative way to the same end would have been to start with a higher energy for the muon beam. For example, the neutrino beam from a 50 GeV storage ring is collimated sufficiently enough to get a similar significance with even the 'low luminosity' option for the storage ring (10^{19} muon decays per year). However, we shall desist from using this option as such a machine is likely only as a second generation facility.

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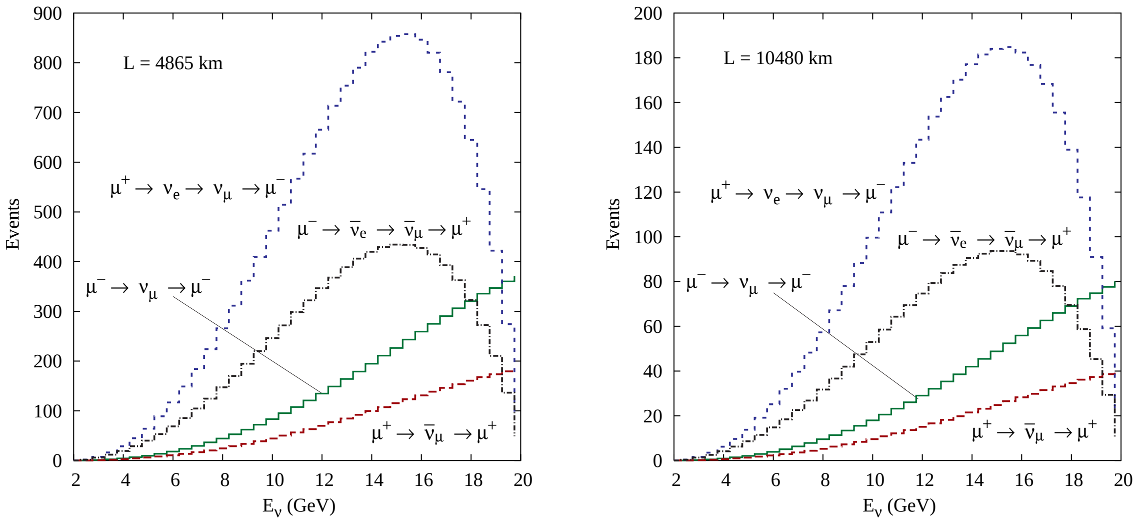

Figure 7: The number of events expected as a function of the (anti-)neutrino energy if the survival probability were to be unity. The source is a 20 GeV muon storage ring with 10^{20} (anti-)muons decaying while the detector is a 50 kT iron calorimeter with a energy threshold of 2 GeV for the $\mu^\pm$ [18]. The left (right) panel corresponds to a baseline of 4865 (10480) km. Also shown are the events generated by the electron (anti-)neutrino assuming a transition probability of unity.

- As figure 4 amply demonstrates, for longer baselines such as the one we are considering, the survival probabilities are very sensitive functions of the (anti-)neutrino energy, especially for low energies. Thus, extracting any information from the low energy tail of the detector muon spectrum would necessitate very good energy resolution. However, with the neutrino-nucleon cross sections being small for such E_ν, these neutrinos have only a relatively small contribution to make to the total number of muon events.

Keeping in mind that $\nu - N$ cross-section is nearly double[6] that of $\bar{\nu} - N$ we argue that the detector should be exposed to a $\bar{\nu}_\mu$ beam for double the time that it is exposed to a ν_μ beam. To be specific, we consider a 50 kT-year exposure to a ν_μ beam and a 100 kT-year exposure to $\bar{\nu}_\mu$ beam, or in other words, a total exposure of 3 years for the detector configuration [18] under consideration.

5.1 The rate asymmetry for $\nu_\mu(\bar{\nu}_\mu)$ initiated events

Armed with the above, we can now calculate the difference in μ^- and μ^+ event numbers. Defining an asymmetry of the form

$$A_N \equiv \frac{N_- - N_+}{N_- + N_+}\,, \tag{5.2}$$

[6] For the total cross section, this ratio has only a mild dependence on the incident neutrino energy, and for the energy range we are interested in has a value close to 2.18. However, on imposition of an energy threshold $E(\mu^\pm) > 2\,\text{GeV}$ that such a detector is expected to have, the ratio is a more sensitive function of the (anti-)neutrino energy (see figure 7). In our numerical analysis, we explicitly account for this factor and the consequent effect on the error bars.

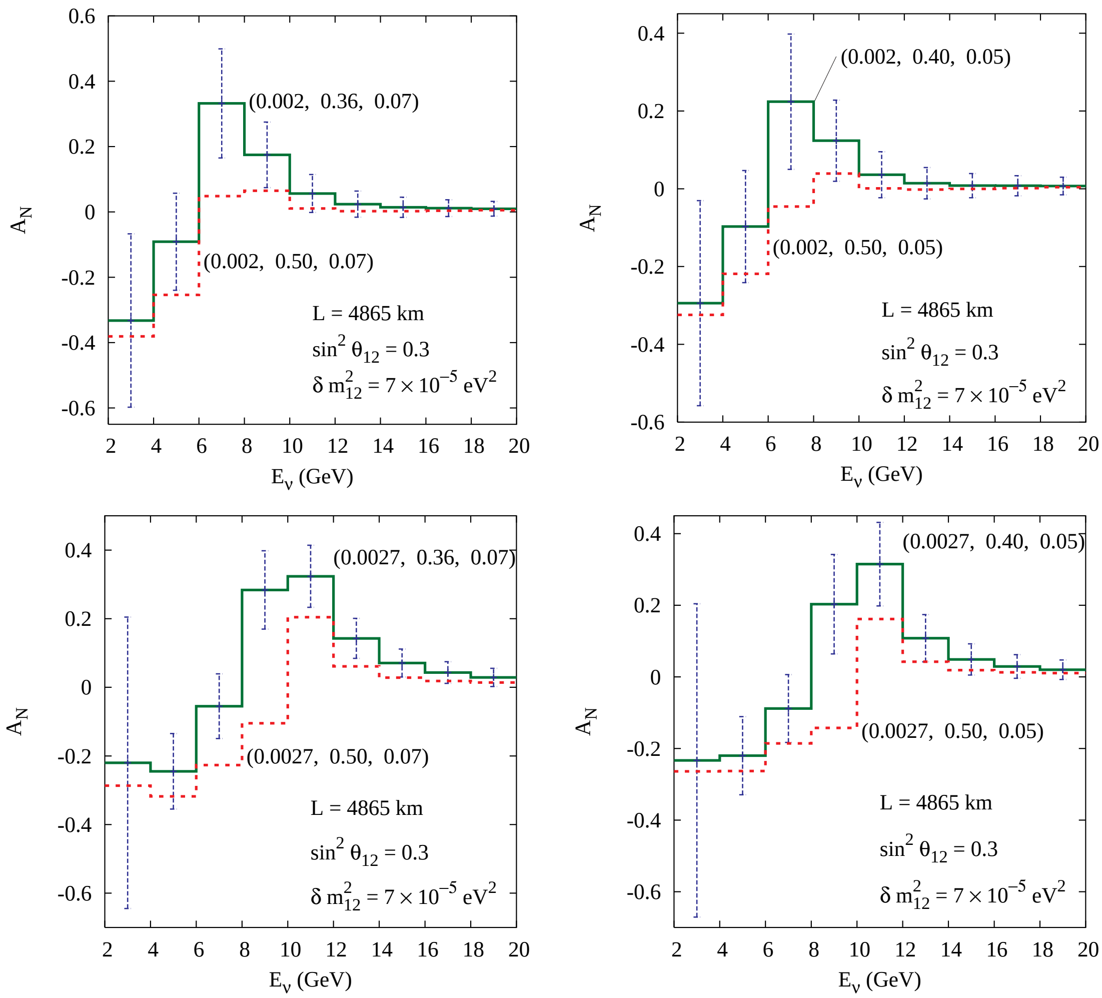

Figure 8: The difference between ν_μ and $\bar{\nu}_\mu$ charged-current events for a baseline $L = 4865$ km with an integrated luminosity of 10^{20} muons and double the number of anti-muons. The detector is a 50 kT iron calorimeter with a muon energy threshold of 2 GeV [18]. Each plot corresponds to a different combination of $(\delta m^2_{13}, |U_{\mu 3}|^2, |U_{e3}|^2)$. The other parameters are: $\delta m^2_{12} = 7 \times 10^{-5}\,\mathrm{eV}^2$ and $\sin^2\theta_{12} = 0.3$. Also shown are the 1σ error bars.

we present, in figure 8, the same for a baseline of 4865 km. The values for $(|U_{\mu 3}|^2, |U_{e3}|^2)$ correspond to the two extreme cases of ref. [20]. Also plotted are the analogous expectations for $|U_{\mu 3}|^2 = 0.5$, a case that, to the lowest order, would have been associated with vanishing A_N. We have binned the expected data, keeping under consideration both the expected energy resolution [18] as well as the number of events in a particular bin. Also shown are the statistical 1σ error bars. Several features are easily discernible:

- In each of the cases displayed, the rate asymmetry is statistically significant in more than one bin. Together, they lead to a very considerable shift in the χ^2. This amply demonstrates the sensitivity of such an experiment to a departure from maximality of $U_{\mu 3}$.

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- As expected, the signal is less pronounced for the case $(|U_{\mu 3}|^2, |U_{e3}|^2) = (0.40, 0.05)$ than for $(0.36, 0.07)$. This is just a reflection of the fact — see eq. (2.9) — that the rate asymmetry is proportional to $|U_{e3}|^2$. While the proportionality to $(1 - 2\,|U_{\mu 3}|^2)$ is no longer an accurate statement, it, nevertheless still encapsulates a large measure of truth.

- In the energy range of interest, the bin wise asymmetry clearly shows an oscillatory behaviour, a feature that would have been absent for a similar detector had the baseline been shorter than ~ 1000 km.

- The functional dependence of $\Delta N(E_\nu)$ exhibits a discernible dependence on the mixing angles $(|U_{\mu 3}|^2, |U_{e3}|^2)$, and could, in principle, be used to distinguish between values for these parameters. However, to obtain a significant resolution in the parameter space, a much larger event count would be necessary.

- The dependence on the value of δm^2_{13} , on the other hand, is much more pronounced. The shift in ΔP that we encountered in figure 4 is quite well reflected by a shift in A_N, even after the convolution with the muon spectrum and the energy-dependent cross sections as well as the finite resolution effects. Thus, our previous comment about using this measurement for a determination of δm^2_{13} is substantiated.

- The energy dependence is quite different for the two baselines considered and can be used to advantage. For example, if storage rings come up at both the JHF [26] and Fermilab, a detector such as ICAL/INO [18] could use beams from both to distinguish more efficiently between the possible parameter sets.

- Finally, a comment on the use of A_N. The use of the ratio (eqn 5.2) rather than just the difference $(N_- - N_+)$ serves to eliminate much of the systematic errors. Note though that A_N, being a ratio of numbers, is not immediately reflective of the actual number of events. Consequently, the apparently large A_N values obtainable for the bins with smaller E_ν values have much larger error bars assciated with them and hence are of relatively low statistical significance.

5.2 Wrong sign muons

We now turn to a discussion of ν_e to ν_μ conversion (as well as the conjugate process) and the subsequent detection of the latter in the iron calorimeter detector under consideration. Such events are obviously characterised by the appearance of a muon with a charge opposite to that of the decaying particle in the storage ring. Since the detector is to be a magnetized one, charge measurement is relatively straightforward thus rendering these events easily distinguishable. Efficacy of 'wrong-sign-muon' events in measuring the angle θ_{13} has already been advocated in refs. [11, 12, 27] in the context of muon-storage ring neutrino factory experiments. Here, we want to emphasise that an independent measurement of 'wrong-sign-muon' events can also supplement the more accurate measurement of θ_{23}. We have already seen, vide figure 6, that the conversion probability for ν_e is typically larger than that for $\bar{\nu}_e$, a consequence of the sign of δm^2_{12}. This fact, alongwith the smaller detection efficiency

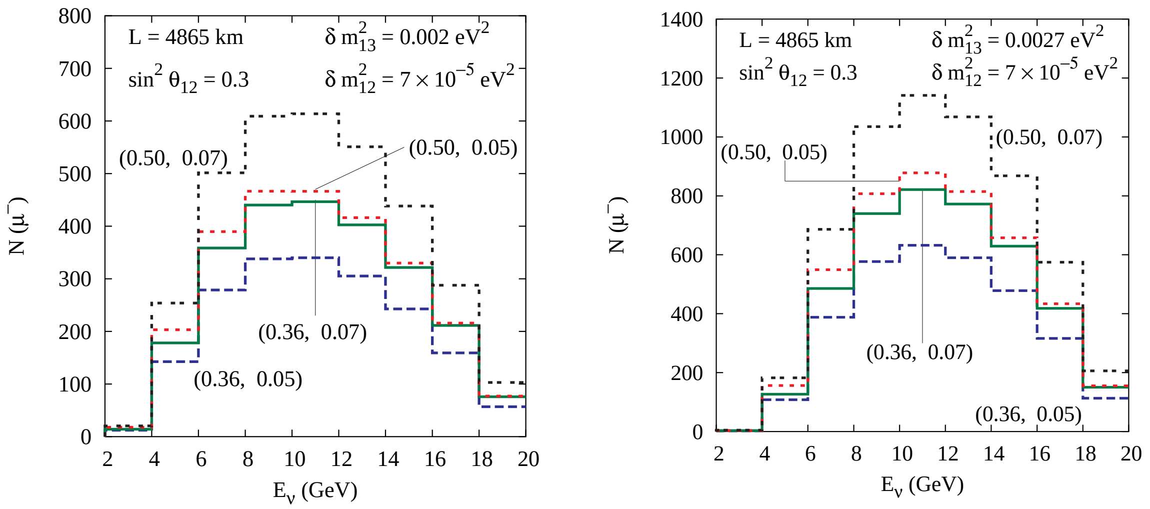

Figure 9: The number of μ^- events in a 50 kT iron calorimeter exposed to a total of 2×10^{20} μ^+ decays in a 20 GeV storage ring at a distance of 4865 km. The histograms correspond to different values of $(|U_{\mu 3}|^2, |U_{e3}|^2)$ with the left (right) panels referring to $\delta m^2_{13} = 0.002\,(0.0027)\,\mathrm{eV}^2$.

for $\bar{\nu}_\mu$, implies that it is wiser to concentrate on the $\mu^+ \to \nu_e \to \nu_\mu$ chain rather than attempt to measure the conjugate process as well. And since we have already advocated twice as long an exposure to μ^+ decays as compared to μ^- decays, this works in our favour too.

In figure 9, we present the number of μ^- events in the detector when exposed to μ^+ decays. As before, the detector is supposed to be a 50 kT one, with a total exposure of 2×10^{20} decays. Clearly, the number of events grows with $|U_{e3}|^2$. While the exact dependence is not linear (as the naive expectation would be), explicit calculations with other values of $|U_{e3}|$ shows that it is not very far from being so. This is also the case for the larger baseline of 10480 km (which we do not exhibit here). The dependence on $|U_{\mu 3}|$ is more complicated though. As can be expected, this depends more crucially on the size of the matter effect and hence on the exact baseline (see, for example, figure 6). However, given the considerably large number of such events, it is quite apparent that this measurement is likely to be a sensitive probe in the $(|U_{\mu 3}|, |U_{e3}|)$ plane. Unfortunately though, the event distribution is not very sensitive to δm^2_{13}.

Finally, we turn to the issue of charge measurement. While charge identification in a magentized detector, such as the MONOLITH [17] or ICAL/INO [18] experiments, is theoretically straightforward, in reality there is always a possibility for misidentification. Thus some of the wrong sign muon events would be identified as right sign ones and vice versa. Denoting the probability of misidentification by $\mathcal{P}$ (in other words, the purity of charge measurement is $1 - \mathcal{P}$), and starting from a beam of decaying μ^-, the number of events in the detector that would be *identified* as containing a μ^- is now given as

$$N^{\mathrm{raw}}_{-;-} = (1 - \mathcal{P})\, N(\mu^- \to \nu_\mu \to \mu^-) + \mathcal{P}\, N(\mu^- \to \bar{\nu}_e \to \bar{\nu}_\mu \to \mu^+) \tag{5.3}$$

where the notation is an obvious one. An analogous expression obtains for $N^{\mathrm{raw}}_{+;+}$, the num-

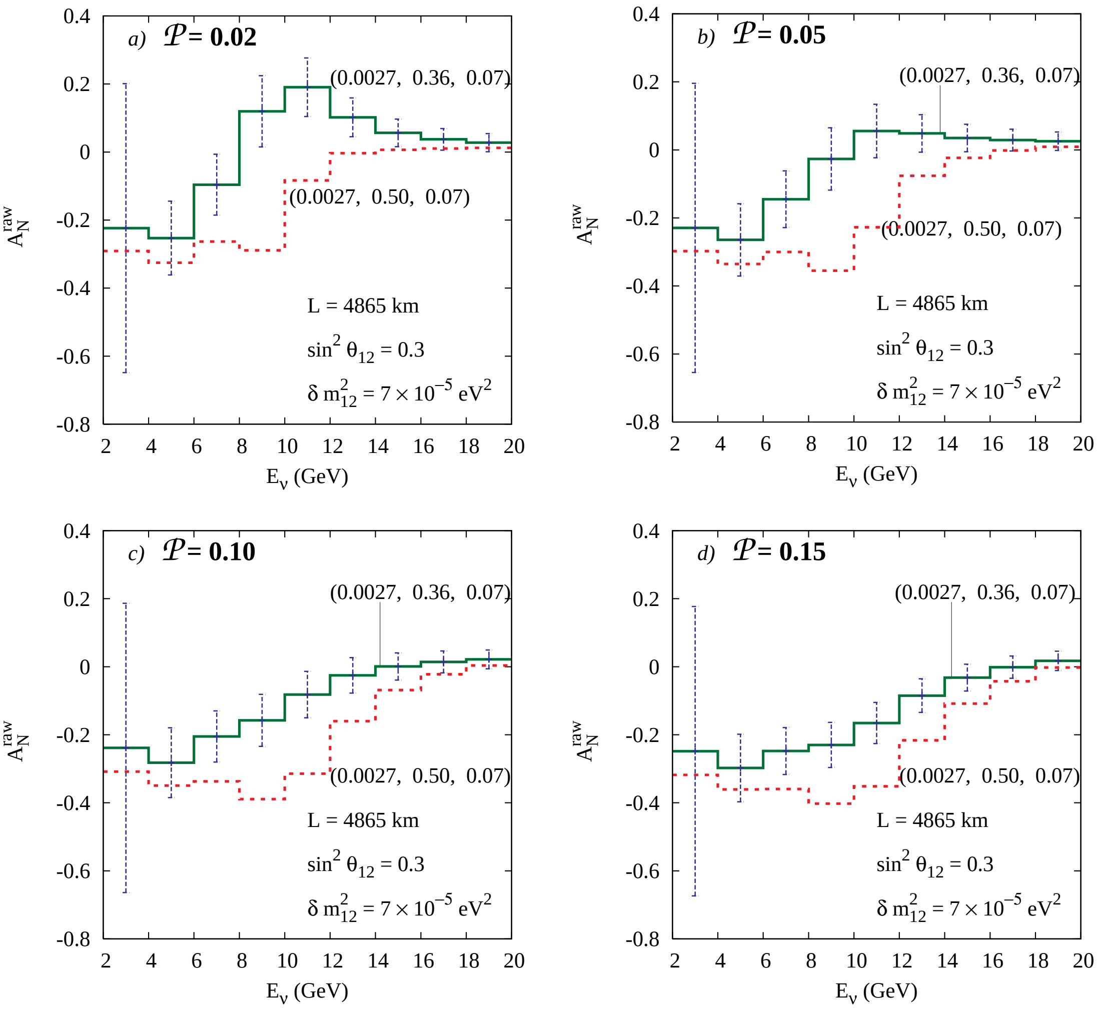

Figure 10: Illustrating the dependence of the difference between ν_μ and $\bar{\nu}_\mu$ charged-current events on the probability of muon charge misidentification $\mathcal{P}$ (see eq. 5.4). Each plot corresponds to a different value of $\mathcal{P}$. The rest of the parameters are as in figure 8, and the ideal case corresponds to figure 8(c).

ber of events classified as μ^+ events for an initial beam of μ^+. The theoretical expression for the event rate asymmetry A_N is now modified to

$$A_N^{\rm raw} \equiv \frac{N^{\rm raw}_{-\,;\,-} - N^{\rm raw}_{+\,;\,+}}{N^{\rm raw}_{-\,;\,-} + N^{\rm raw}_{+\,;\,+}} \,. \tag{5.4}$$

Note that since $N(\mu^+ \to \nu_e \to \nu_\mu \to \mu^-)$ is typically larger than the rate for the conjugate process, the main effect on $N^{\rm raw}_{-\,;\,-}$ is just a relative scaling by the factor $(1 - \mathcal{P})$, whereas $N^{\rm raw}_{+\,;\,+}$ receives a more nontrivial correction. In figure 10, we demonstrate the effect of non-zero values of $\mathcal{P}$ on $A_N^{\rm raw}$. Whereas one expects $\mathcal{P} \lesssim 0.07$ even in the first phase of the ICAL detector [19] for very high efficiencies, and likely to be significantly better at later stages, we, nonetheless, investigate until a higher value, namely $\mathcal{P} = 0.15$. As figure 10 immediately shows, $A_N^{\rm raw}$ continues to be a robust measurement of the deviation from maximality, even in the face of significant impurity in muon charge identification.[7]

6. Conclusions

Analyses of matter effects in ν_μ oscillations have largely concentrated on transition probabilities. Revisiting the problem, we demonstrate that the survival probabilities too are very sensitive to matter effects and can be used profitably in determination of several crucial parameters in the neutrino sector.

To start with, we derive a set of approximate analytical expressions for the ν_μ survival probability in presence an arbitrarily large matter density. While similar results had already been reported for small matter terms in the effective hamiltonian, our expressions have a much wider range of validity.

Since we are interested in the survival probability for ν_μ, a variable of interest can be constructed simply from the number of charged current (CC) events in the detector. Starting with ν_μ and $\bar{\nu}_\mu$ beams, $A_N \equiv (N_- - N_+)/(N_- + N_+)$ describes an asymmetry between the number of 'same-sign' μ^- and μ^+ events generated in the detector after traversing a given baseline. We show, through an explicit calculation of event rates, that this asymmetry is a good measure of the matter effects felt by the neutrinos while propagating through the earth. However, contrary to claims in the literature [20], the size of the asymmetry for the Fermilab-MINOS combine is much smaller than the experimental sensitivity.

We suggest, therefore, that a future long-baseline experiment could explore this effect. As a prototype experiment, we consider a neutrino factory (a muon storage with muon beams of energy 20 GeV) and the proposed 50 kT iron calorimeter detector [17, 18] with a capability of muon charge determination. Using realistic experimental resolutions, we demonstrate that such a combination is sufficient to establish the aforementioned asymmetry with a very large statistical significance. However, unlike in the case of relatively short-baseline Fermilab-MINOS combine, the asymmetry for a long baseline is no longer proportional to the deviation of $|U_{\mu 3}|^2$ from maximality. Even so, the sensitivity to $(1-2\,|U_{\mu 3}|^2)$ remains quite pronounced for baselines of upto about 6500 Km. Using the aforementioned approximate expressions for the survival probabilities, such data can thus be used to determine $(1-2\,|U_{\mu 3}|^2)$. While the sensitivity to this combination decreases for much longer baselines, asymmetry data for such baselines can still be used for precise determination of $|U_{e3}|$.

The main motivation for the above exercise is to show that the measurement of any of the small parameters ($|U_{e3}|$ or β) at long baseline experiments cannot be done independent of each other. Although experiments with relatively small baselines (such as NuMi-MINOS) seem to be quite sensitive to the parameter β, this optimism is misplaced as explained in sections 3 & 4. On the other hand, moderate baseline experiments do show increased sensitivity to β, but only at the cost of having a more complicated dependence on β and $|U_{e3}|$ (see eq. 2.14 and figure 5). In either case, a measurement of β from ν_μ-survival

[7]Although the simulations presented here are valid only for an energy independent $\mathcal{P}$, the inclusion of an energy dependent impurity would not materially affect the conclusions.

probability asymmetry needs an accurate knowledge of $|U_{e3}|$. It is here that the wrong-sign muon rates (which is more sensitive to U_{e3}) have the most important role to play. Of course, a precise knowledge of δm^2_{13} is important for the extraction of $|U_{e3}|$ (although the dependence on δm^2_{13} is not as pronounced for the wrong-sign muon events as for the right-sign ones). However, it is expected that the error-bars on δm^2_{13} would be significantly reduced before the era of such neutrino factory experiments. Then, in principle, A_N and wrong-sign muon rates could be used simultaneously in a statistical analysis (in a spirit similar to that followed in ref. [13]) to extract $|U_{e3}|$ and β to much greater accuracy than possible today.

Acknowledgments

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References

[1] SUPER-KAMIOKANDE collaboration, Y. Hayato, talk given at *EPS 2003 conference*, Aachen, Germany 2003, http://eps2003.physik.rwth-aachen.de. SUPER-KAMIOKANDE collaboration, S. Fukuda et al., *Determination of solar neutrino oscillation parameters using 1496 days of super-kamiokande-i data*, *Phys. Lett.* **B 539** (2002) 179 [hep-ex/0205075].

[2] SNO collaboration, S.N. Ahmed et al., *Measurement of the total active b-8 solar neutrino flux at the sudbury neutrino observatory with enhanced neutral current sensitivity*, *Phys. Rev. Lett.* **92** (2004) 181301 [nucl-ex/0309004].

[3] M. Apollonio et al., *Search for neutrino oscillations on a long base-line at the chooz nuclear power station*, *Eur. Phys. J.* **C 27** (2003) 331 [hep-ex/0301017].

[4] KAMLAND collaboration, K. Eguchi et al., *First results from kamland: evidence for reactor anti-neutrino disappearance*, *Phys. Rev. Lett.* **90** (2003) 021802 [hep-ex/0212021].

[5] K2K collaboration, I. Kato, *The results of oscillation analysis in K2K experiment and an overview of JHF-ν experiment*,talk given at *38th Recontres de Moriond on Electroweak Interactions and Unified Theories*, Les Ares, France 2003 [hep-ex/0306043].

[6] For a recent review for the limits on masses and mixing, see S. Goswami, A. Bandyopadhyay and S. Choubey, *Global analysis of neutrino oscillation*, *Nucl. Phys.* **143** *(Proc. Suppl.)* (2005) 121 [hep-ph/0409224].

[7] L. Wolfenstein, *Neutrino oscillations in matter*, *Phys. Rev.* **D 17** (1978) 2369;
S.P. Mikheev and A.Y. Smirnov, *Resonance enhancement of oscillations in matter and solar neutrino spectroscopy*, *Sov. J. Nucl. Phys.* **42** (1985) 913.

[8] For example, see S. Geer, *Neutrino factory: physics and R & D status*, FERMILAB-CONF-04-133-E, 2004;
R. Raja, *Status of neutrino factory R & D within the muon collaboration*, hep-ex/0402022;
A. Blondel et. al., *The neutrino factory: beam and experiments*, *Nucl. Instrum. Meth.* **A451** (2000) 102.

[9] V. Barger, S. Geer, R. Raja and K. Whisnanat, *Long baseline study of the leading neutrino oscillation at a neutrino factory*, *Phys. Rev.* **D 62** (2000) 013004;
C. Albright et al., *Physics at a Neutrino Factory*, *Physics at a neutrino factory*, hep-ex/0008064;
M. Freund, P. Huber and M. Lindner, *Systematic exploration of the neutrino factory parameter space including errors and correlations*, *Nucl. Phys.* **B 615** (2001) 331;
M. Apollonio et al., *Oscillation physics with a neutrino factory*, hep-ph/0210192.

[10] M. Freund, P. Huber and M. Lindner, *Extracting matter effects, masses and mixings at a neutrino factory*, *Nucl. Phys.* **B 585** (2000) 105.

[11] V.D. Barger, S. Geer, R. Raja and K. Whisnant, *Determination of the pattern of neutrino masses at a neutrino factory*, *Phys. Lett.* **B 485** (2000) 379 [hep-ph/0004208].

[12] A. Cervera et al., *Golden measurements at a neutrino factory*, *Nucl. Phys.* **B 579** (2000) 17, erratum *ibid.* **B593** (2001) 731.

[13] M. Aoki et al., *Prospects of very long baseline neutrino oscillation experiments with the KEK/JAERI high intensity proton accelerator*, *Phys. Rev.* **D 67** (2003) 093004;
M. Aoki, K. Hagiwara and N. Okamura, *Lifting degeneracies in the oscillation parameters by a neutrino factory*, *Phys. Lett.* **B 606** (2005) 371 [hep-ph/0311324].

[14] S. Antusch *et al.*, *Is there maximal mixing in the lepton sector?*, hep-ph/0404268.

[15] D. Indumathi and M.V.N. Murthy, *A question of hierarchy: matter effects with atmospheric neutrinos and anti-neutrinos*, *Phys. Rev.* **D 71** (2005) 013001 [hep-ph/0407336].

[16] I. Mocioiu and R. Shrock, *Phys. Rev.* **D 62** (200) 053107;
R. Gandhi, P. Ghoshal, S. Goswami, P. Mehta and S. Uma Sankar, *Large matter effects in $\nu_\mu \to \nu/\tau$ oscillations*, *Phys. Rev. Lett.* **94** (2005) 051801 [hep-ph/0408361].

[17] MONOLITH collaboration, N.Y. Agafonova et al., *Monolith: a massive magnetized iron detector for neutrino oscillation studies*, LNGS-P26-2000, LNGS-P26-00, CERN-SPSC-2000-031, CERN-SPSC-M-657;
Also see http://castore.mi.infn.it/~monolith/

[18] See http://www.imsc.res.in/~ino and working reports and talks therein.

[19] A. Raychaudhuri, private communication.

[20] S. Choubey and P. Roy, *Testing maximality in muon neutrino flavor mixing*, *Phys. Rev. Lett.* **93** (2004) 021803 [hep-ph/0310316].

[21] As for example see, J. Arafune, M. Koike and J. Sato, *CP violation and matter effect in long baseline neutrino oscillation experiments*, *Phys. Rev.* **D 56** (1997) 3093 [hep-ph/9703351], erratum *ibid.* **D60** (1999) 119905.

[22] A.M. Dziewonski and D.L. Anderson, *Phys. Earth Plan. Int.* **25** (1981) 297;
We use the parametrisation given in R. Gandhi, C. Quigg, M. Reno and I. Sarcevic, *Ultrahigh-energy neutrino interactions*, *Astropart. Phys.* **5** (1996) 81.

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[23] E. Akhmedov et al., *Series expansions for three-flavor neutrino oscillation probabilities in matter*, *JHEP* **04** (2004) 078.

[24] MINOS collaboration, R. Saakian, *Status of the minos experiment*, *Nucl. Phys.* **111** *(Proc. Suppl.)* (2002) 169; *Status and prospects of the minos experiment*, *Yad. Fiz.* **67** (2004) 1112; M.V. Diwan, *Status of the minos experiment*, *eConf* **C0209101** (2002) TH08 [hep-ex/0211026].

[25] MINERνA collaboration, D.A. Harris et al., *Neutrino scattering uncertainties and their role in long baseline oscillation experiments*, hep-ex/0410005.

[26] Y. Itow et al., *The JHF-kamioka neutrino project*, hep-ex/0106019; also see: http://neutrino.kek.jp/jhfnu.

[27] P. Huber, T. Schwetz and J. Valle, *How sensitive is a neutrino factory to the angle θ_{13}?*, *Phys. Rev. Lett.* **88** (2002) 101804.